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A Simple Formula for the Gilliland Correlation in Multicomponent Distillation

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Abstract

Past attempts to represent the Gilliland correlation by an analytic expression were not satisfactory. Some authors found it necessary to divide the span into two or three sections with a different equation for each section. The only equation in the literature which covers the whole span is quite complicated yet it has been reproduced in well-known textbooks of chemical engineering. A much simpler equation which should be just as accurate is given in this paper. The Gilliland correlation has been plotted in the literature either on regular or on log-log graph paper. In this paper a linear plot of the correlation has been achieved by plotting it on a specially prepared graph paper where the ordinate is linear and the abscissa is nonlinear. The distances at which the graduations are marked on the x-axis were obtained from a derived equation.

INTRODUCTION

The well-known short-cut method for calculating the number of theoretical plates in multicomponent distillations is based on calculating the minimum number of theoretical plates (N_{min}) at total reflux using the Fenske equation and the minimum reflux ratio (R_{min}) using the Underwood method. From N_{min} and R_{min} the number of theoretical plates corresponding to a reflux ratio R may be calculated using the Gilliland correlation (*1*) or the Erbar-Maddox correlation (*2*).

The Gilliland correlation is simple enough and is quite useful for preliminary design calculations. The results obtained from it are sufficiently accurate for certain multicomponent distillation calculations. When more accuracy is required, stage-to-stage calculations using more

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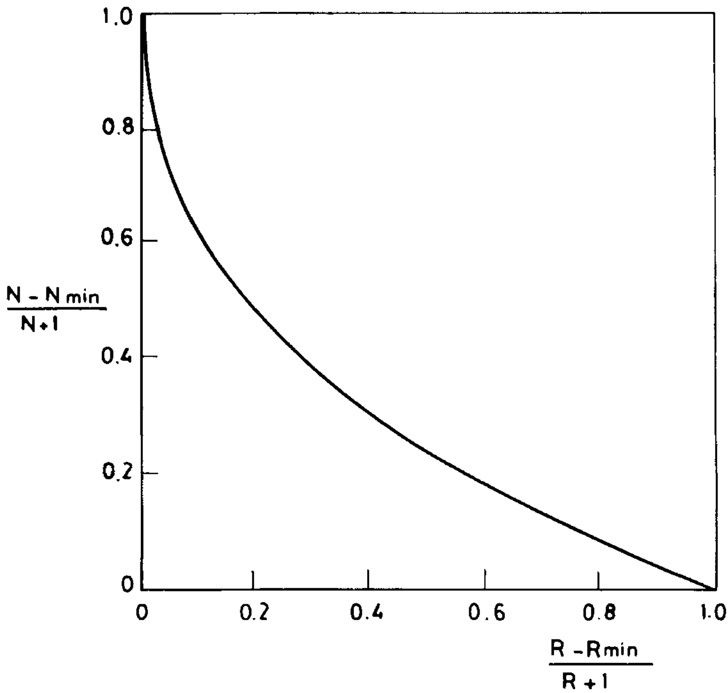

FIG. 1. The Gilliland correlation with linear coordinates.

rigorous methods are utilized and a high-speed computer becomes essential for executing the calculations.

The Gilliland correlation, introduced in 1940, is shown in Fig. 1 where $(N - N_{min})/(N + 1)$ is plotted versus $(R - R_{min})/(R + 1)$. This curve represents the best fit of a collection of experimental data. Some data may deviate from this curve by $\pm 5\%$ or more. For simplicity we define Y and X as follows:

$$Y = \frac{N - N_{min}}{N + 1}, \qquad X = \frac{R - R_{min}}{R + 1}$$

As can be seen from Fig. 1, the curve has a slope equal to $-\infty$ at $X = 0$ and a finite slope at $X = 1$. Values of X of practical interest lie between $X = 0.1$ and $X = 0.4$, and very seldom fall below 0.05 or exceed 0.5.

Attempts to represent the entire Gilliland curve by an analytic expression (*3, 4*) did not satisfy the boundary conditions of the curve, and the authors had to divide the curve into two or three segments with a different equation for each segment.

In 1972, Molokanov (*5*) introduced an equation for the whole curve $Y = f(X)$ between $X = 0$ and $X = 1$. The Molokanov equation reads

$$Y = 1 - \exp\left(\frac{1 + 54.4X}{11 + 117.2X}\right)\left(\frac{X - 1}{\sqrt{X}}\right) \quad (1)$$

Equation (1) has been presented in recent textbooks on chemical engineering; for example, Henley and Seader's book on stage separation operations (*6*).

Values of Y corresponding to different values of X were calculated from Eq. (1). They are listed in Column 2 of Table 1.

The first impression one gets from inspecting the Molokanov formula is that it is too complex for representing a curve subject to such significant uncertainties and where the emphasis is on a short segment of the curve, namely that portion between $X = 0.1$ and $X = 0.4$. It was felt that a much simpler expression should be capable of representing the Gilliland curve satisfactorily.

TABLE 1
Comparison between Suggested Equations and Molokanov Equation

X	Y, Eq. (1), Molokanov	Y, Eq. (7), fitting one point and slope	Y, Eq. (10), fitting two points
0	1	1	1
0.05	0.607	0.658	0.625
0.1	0.553	0.573	0.553
0.15	0.504	0.511	0.500
0.2	0.460	0.461	0.455
0.25	0.418	0.417	0.416
0.3	0.380	0.377	0.380
0.35	0.344	0.341	0.346
0.4	0.311	0.307	0.315
0.5	0.249	0.246	0.256
0.6	0.192	0.190	0.201
0.7	0.139	0.139	0.148
0.8	0.090	0.090	0.098
0.9	0.044	0.044	0.048
1.0	0	0	0

THE SIMPLE ALTERNATIVE FORMULA

The authors suggest as an alternative to Eq. (1), a simple equation in the form

$$Y = \frac{1 - X^n}{1 - aX^n} \tag{2}$$

where a and n are constants.

By differentiating, one can show that both Curves (1) and (2) have slopes equal to $-\infty$ at $X = 0$. Furthermore, at $X = 1$:

$$\text{Slope of Curve (1)} = -0.43 \tag{3}$$

$$\text{Slope of Curve (2)} = \frac{n}{a - 1} \tag{4}$$

Assuming that the Molokanov formula gives a good fit for the Gilliland correlation, constants a and n are obtained by fitting Curve (2) as close as possible to Curve (1), particularly in the region of practical interest. This can be accomplished by two methods:

1. Equating the two slopes at $X = 1$ and the two ordinates at a point in the middle of the region of practial applicability, say at $X = 0.2$.
2. Equating the two ordinates at two points in the region of interest, say at $X = 0.1$ and 0.3.

Each of these methods leads to two equations which can be solved easily for the two unknowns a and n.

First Method. From Eqs. (3) and (4) and values listed in Table 1, we get

$$\frac{n}{a - 1} = -0.43 \tag{5}$$

$$0.460 = \frac{1 - 0.2^n}{1 - a(0.2^n)} \tag{6}$$

Solving for n and a gives, after minor adjustments, the following values: $a = 0.42$ and $n = 0.25$, and therefore

$$Y = \frac{1 - X^{0.25}}{1 - 0.42X^{0.25}} \tag{7}$$

Solving for X leads to

$$X = \left(\frac{1 - Y}{1 - 0.42Y}\right)^{4} \tag{7a}$$

Second Method. Again from values listed in Table 1, one gets:

$$0.553 = \frac{1 - 0.1^{n}}{1 - a(0.1^{n})} \tag{8}$$

$$0.380 = \frac{1 - 0.3^{n}}{1 - a(0.3^{n})} \tag{9}$$

Solving Eqs. (8) and (9) for a and n leads, after minor adjustments, to the following values: $a = 0.8$ and $n = 0.096$, and hence

$$Y = \frac{1 - X^{0.096}}{1 - 0.8X^{0.096}} \tag{10}$$

Solving for X we get

$$X = \left(\frac{1 - Y}{1 - 0.8Y}\right)^{10.42} \tag{10a}$$

Equations (7a) and (10a) are explicit relations for X in terms of Y. An explicit expression in x based on Eq. (1) is not possible.

Equations (7) and (10) are tabulated in Table 1 in Columns 3 and 4, respectively, together with Eq. (1) for comparison.

As expected, Table 1 shows that Eq. (7) gives an accurate fit at values of X greater than 0.5, which is not the important part of the curve, while the fit in the important region is only reasonably good with a maximum error which might not be within the uncertainty limit of Eq. (1).

On the contrary, Eq. (10) gives an approximate fit in the less important region of the curve and an accurate fit between $X = 0.1$ and $X = 0.4$ with a maximum absolute error equal to 0.005, which should be well within the uncertainty limits of Eq. (1). Equation (10) is therefore a better equation for expressing the relation between Y and X in the Gilliland correlation.

LINEAR PLOT OF THE GILLILAND CORRELATION

The Gilliland correlation is presented in the literature as a curve on either regular graph paper or log-log graph paper. A more convenient plot would be a linear one, and this requires preparing a special kind of graph paper where the ordinate is a linear scale and the abscissa is nonlinear. The relation between the distance l on the nonlinear scale and the markings of X is deduced as follows: If the total length of the x-axis is L and the distance on it from its beginning at the left is l, then for the function $Y = f(x)$ to be represented by a straight line, Y should vary linearly with l so that

$$Y = A + B'l \tag{11}$$

A and B' are constants. Let

$$Z = l/L = \text{length fraction}$$

hence

$$Y = A + B'LZ$$

or

$$Y = A + BZ$$

and since

$$Y = f(x)$$

then

$$A + BZ = f(x)$$

which gives

$$Z = \frac{f(x) - A}{B} \tag{12}$$

The values of the constants A and B are calculated from the boundary conditions of the graph. Assuming that we are interested only in values of X from 0.01 to 0.9, then

$$Z = 0 \text{ or } l = 0 \text{ corresponds to } X = 0.01$$

$$Z = 1 \text{ or } l = L \text{ corresponds to } X = 0.9$$

Substituting these values in Eq. (12) and utilizing the Molokanov expression for $f(x)$ leads to

$$A = 0.715$$

$$B = -0.671$$

and therefore

$$Z = \frac{0.715 - f(x)}{0.671} \tag{13}$$

taking $L = 20$ cm gives

$$l = 29.81(0.715 - f(x)) \tag{14}$$

Values of l corresponding to key values of X were calculated from Eq. (14) and are listed in Table 2. These values were utilized in locating the X markings on the horizontal axis.

Since the plot is a straight line, only two points on it are needed. At $l = 0$, $X = 0.01$. Substituting $X = 0.01$ in Eq. (1) gives

$$Y = 0.715$$

Similarly by substituting $X = 0.9$ in Eq. (1) gives

$$Y = 0.044$$

TABLE 2
Distance l on the Horizontal Coordinate of Fig. 2 for Different Values of X

X	l (cm)	X	l (cm)	X	l (cm)	X	l (cm)
0.01	0	0.06	3.58	0.15	6.28	0.5	13.90
0.02	1.68	0.07	3.90	0.2	7.62	0.6	15.60
0.03	2.36	0.08	4.24	0.25	8.84	0.7	17.16
0.04	2.82	0.09	4.54	0.3	9.98	0.8	18.62
0.05	3.22	0.1	4.84	0.4	12.06	0.9	20.00

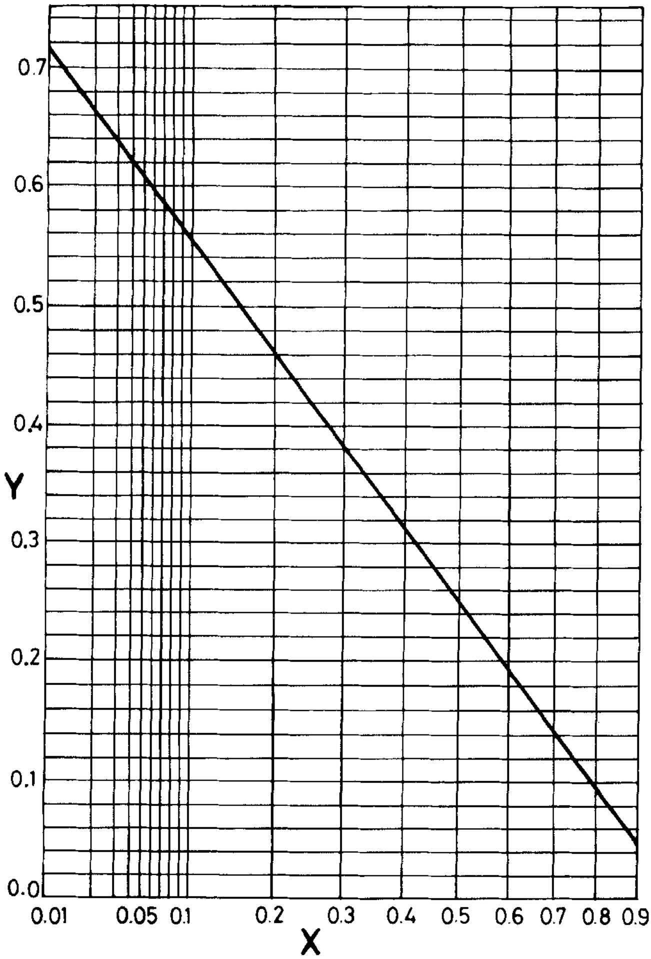

FIG. 2. A linear plot of the Gilliland correlation.

The straight line is drawn on the graph by connecting the two points ($l = 0$, $Y = 0.715$) and ($l = 20$, $Y = 0.044$) as shown in Fig. 2.

The Molokanov equation equation was utilized in the preparation of the chart not because of its superiority, but because such a chart would reduce considerably the effort of calculating X corresponding to a given Y. For this, trial and error is needed in the case of the Molokanov equation. A similar chart based on the same principles can be prepared for Eq. (10) suggested by the present authors. Such a chart is really not necessary due to the simplicity of this equation. Furthermore, no trial and error is required for calculating either Y or X.

FITTING THE CORRELATION WITH A ONE-CONSTANT EQUATION

One can also think of several one-constant equations which have the same boundary conditions as the Gilliland correlation. The following two equations belong to this group:

$$Y = 1 - X^n \tag{15}$$

where n is a positive fraction, and

$$Y = 1 - c\sqrt{X} + (c - 1)X \tag{16}$$

where c is a positive constant. Both functions are equal to 1 at $X = 0$ and equal to 0 at $X = 1$. Also

$$(dY/dX)_{X=0} = -\infty$$

and $(dY/dX)_{X=1}$ has a finite value equal to $-n$ for Eq. (15) and $(\frac{1}{2}c - 1)$ for Eq. (16).

The constants n and c were calculated at different key points and are tabulated in Table 3. The key point is the point at which the original and fitting functions are made to coincide. X_{KP} is the X coordinate of the key point.

Table 3 shows that the value of n changes somewhat between $X_{KP} = 0.1$ and $X_{KP} = 0.3$, indicating that Eq. (15) gives only an approximate fit for the Gilliland correlation, while in the same X_{KP} region, c changes appreciably, indicating that Eq. (16) leads to a poor fit of the correlation.

TABLE 3
Values of the Constants n and c in Eqs. (15) and (16) for Different Key Points

Key point			
X_{KP}	Y_{KP}	n	c
0.1	0.553	0.350	1.605
0.2	0.460	0.383	1.375
0.3	0.380	0.397	1.292

TABLE 4
Comparison between the Accuracy of Eqs. (10) and (15)

		$E = Y - Y_m$				
			Eq. (15)			
X	Y_m (Eq. 1)	Eq. (10)	$X_{KP} = 0.1$ $n = 0.350$	$X_{KP} = 0.2$ $n = 0.383$	$X_{KP} = 0.3$ $n = 0.397$	$X_{KP} = 0.33$ $n = 0.4$
0	1	0	0	0	0	0
0.05	0.067	0.018	0.042	0.076	0.089	0.091
0.1	0.553	0	0	0.033	0.046	0.049
0.15	0.504	−0.004	−0.019	0.012	0.025	0.028
0.2	0.460	−0.005	−0.029	0	0.012	0.015
0.25	0.418	−0.002	−0.034	−0.006	0.005	0.007
0.3	0.380	0	−0.036	−0.011	0	0.002
0.35	0.344	0.002	−0.037	−0.013	−0.003	−0.001
0.4	0.311	0.004	−0.037	−0.015	−0.006	−0.004
0.5	0.249	0.007	−0.034	−0.016	−0.008	−0.007
0.7	0.139	0.009	−0.022	−0.011	−0.007	−0.006
1	0	0	0	0	0	0

Furthermore, an explicit relation in X based on this equation is relatively not simple and hence Eq. (16) is not a suitable equation for fitting the Gilliland correlation.

Table 4 compares the accuracies of Eqs. (10) and (15) where Y_m is the Y coordinate of Eq. (1). The error $E = Y - Y_m$ is tabulated for both functions at different values of X.

As can be seen from Table 4, the maximum absolute error $|E|$ on using Eq. (10) in the region of practical interest is 0.005. The table also shows that Eq. (15) does not cover the whole region from $X = 0.1$ to $X = 0.4$ adequately, yet when $n = 0.4$ the corresponding simple relation gives a good approximation of the Gilliland correlation for values of X above 0.25 and therefore

$$Y = 1 - X^{0.4}, \qquad X > 0.25 \tag{17}$$

CONCLUSIONS AND SUGGESTIONS

Equation (10) gives an accurate estimate of the Gilliland correlation throughout the range from $X = 0$ to $X = 1$, and particularly in the range of practical interest between $X = 0.1$ and $X = 0.4$. It is therefore the equation of choice to replace the less convenient and unnecessarily complicated Molokanov Eq. (1).

For relatively large values of R/R_m which correspond to values of X greater than 0.25, the simple Eq. (17) gives a good approximation of the correlation.

It is suggested that the Gilliland data be replotted such that $(N - N_{min})/N$ is plotted versus $(R - R_{min})/R$ instead of the original parameters. This is even more logical and may lead to a curve which can be accurately approximated by the simple Eq. (15) throughout the whole region of practical interest. It seems that including +1 in the denominator of the two parameters has contributed to the large curvature in the Gilliland correlation below $X = 0.2$ which cannot be satisfied by such simple equations as Eq. (15).

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